Active Timber Management by Outsourcing Stumpage Price Uncertainty

with the American Put Option

Sun Joseph Chang

School of Renewable Natural Resources

Louisianan State University Agricultural Center

Baton Rouge, LA 70803-6202

e-mail: [xp2610@lsu.edu](mailto:xp2610@lsu.edu)

and

Fan Zhang

NielsenIQ

Chicago, IL, 60606

*Abstract*

Timber production is risky, with price uncertainties caused by stumpage price fluctuations. In this study, the American put option was employed to outsource such uncertainty and establish the reservation prices at various stand ages. Put option values and the reservation prices derived thereof, thus, help forestland owners decide whether the stumpage price is high enough to justify an immediate timber harvest. With a series of simulations, we demonstrated that using an American put option to cover the downside price uncertainty could substantially increase the average land expectation value compared to the certainty case. To further enhance the financial outcome, we proposed a rolling put option method that allows forestland owners to seek price uncertainty protection through a series of partial length put options. Overall, with the help of American put option, forestland owners for the first time could actively set the reservation prices in making their harvest decisions, instead of being a price taker like they have always been.

*Introduction*

Timber production is inherently a risky business. Stumpage price fluctuates all the time, creating uncertainty for forestland owners and managers. Over the years, rotation age determination under price uncertainty has remained one of the key topics in forest management and economics. To overcome such uncertainty, over the last 40 years, various analytical techniques have been applied to address this problem. Among them, stochastic dynamic programming has received the most attention.

As one such application, Brazee and Mendelsohn (1988), hereafter B&M, devised the reservation price method to determine both the value of the land and trees as well as the rotation age. As Plantinga (1998) pointed out, the reservation price of B&M method represents a mechanism for incorporating the option value in determining the optimal rotation length**.**  Since the publication of B&M in 1988 and the dissertation by Lohmander (1987), applications of the stochastic dynamic programming technique include Clarke and Reed (1989), Haight and Holmes (1991), Insley and Wirjanto (2010), Lu and Gong (2003), Mohammadi Limaei and Mohammadi (2021), Rakotoarison and Loisel (2017), Thorsen (1999), and Yin and Newman (1997). In addition, articles employing other analytical techniques by Alvarez and Koskela (2007), Chladná (2007), Insley (2002), Morck et al. (1989), Sodal (2002) and Willassen (1998) have been published to address forest management under price uncertainty. Still further, applications of numerical methods include articles such as Duku-Kaakyrie and Nanang (2004), Gjolberg and Guttormen (2002), Hughes (2000), Insley (2002), Insley and Rollins (2005), Khajuria et al. (2012), Manley and Niquidet (2010), Mei and Clutter (2013), Petrasek and Perez-Garcia (2010), Rocha et al. (2006), and Thomson (1991, 1992). Lastly, over the last 12 years three extensive reviews of such applications have been published by Hildebrandt and Knoke (2011), Yousefpour et al. (2012) and Chaudhari et al. (2016). For details of the various applications, readers are referred to these three reviews. All the publications mentioned above, however, tried to internalize the effects of price uncertainty. Forestland owners, thus, remain as passive price takers in the stumpage market.

In this study, we outsourced the price uncertainty with an American put option and applied the binominal tree method of Cox et al. (1979) to directly establish the option value at various stand ages first and then determine their companion reservation prices. Further, rolling put options of different lengths were employed to determine the reservation prices for shorter durations. By picking a specific strike price, the length for the put option, and the ensuing reservation prices at various ages, forestland owners become active price setters in the stumpage market. Rather than merely waiting for a good price in the future, they would leverage American put options to actively manage downside price uncertainty.

Conceptually, the forestland owner of a put option has the choice but not the obligation of selling a specified amount of timber for a specific price (strike price) by a particular date. An American put option allows its owner to exercise it any time over its duration while the European put option can only be exercised at the end of the contract. The American put option was chosen in this study because such a purchase eliminates stumpage price uncertainty over its contract duration. More importantly, the derived option values enabled the forestland owners/managers to establish the reservation prices at various stand ages. Similarly, the reservation prices are established with the rolling put options of shorter durations.

When the stumpage price of the spot market exceeds the reservation price, the stand should be harvested for superior financial returns. In doing so, timber production becomes an active venture with forestland owners selecting the length of the rolling put option, the starting and ending year of the management window, and the strike price. By choosing a specific set of reservation prices appropriate for their level of uncertainty tolerance, forestland owners determine the corresponding rotation age and the land expectation value accordingly.

In its simplest form, at age τ1, τ1 < T1,the forestland owner out-sources stumpage price uncertainty with an American put option O(P1(T1), τ1) with the specified strike stumpage price P1(T1) by the expiration age T1, for a stand volume Q1(T1). In this study, the put option would start at age τ1=15, for an expiration age of T1=70 while the harvest age t1 could be somewhere between 15 and 70. To provide enough time to realize the benefit of the put option, T1 is more than twice the optimal rotation age under certainty (Brazee and Mendelsohn 1988). But since the longer the T1-τ1 is, the more expensive the put option would be. Further, the higher the specified strike price P1(T1) and the higher the stand volume Q1(T1), the costlier the put option would be. Therefore, judicious selections of these variables for the put option represent important management decisions in establishing the reservation prices. In the sections below, the results of applying the American put option to outsource price uncertainty is compared with that of the Brazee and Mendelsohn method. The purposes are to 1. Establish the efficacy of outsourcing stumpage price uncertainty with the American put option, and 2. To explore possible modifications of the American put option method to enhance its results.

*Simulations and Analyses*

In this section, the results of the reservation prices established with the American put option and that by B&M approach were determined and compared. Once the reservation prices were established at various stand ages, simulations were carried out to determine the average rotation age and LEV1 -- the land expectation value at the beginning of the first rotation under the generalized Faustmann formula (Chang 1998).

(1)

Where

V1(t1) =P1(t1)Q1(t1) represents the stumpage value of a t1-year old stand per hectare with P1(t1) being the stumpage price of the t1-year old stand and Q1(t1) being the stand volume per hectare of the t1-year old stand.

C1 represents the regeneration cost per hectare of the first rotation.

r1 represents the real annual interest rate during the first rotation.

LEV2 represents the land expectation value per hectare at the end of the first rotation and the beginning of the second rotation. It embodies all the optimal rotation ages of future rotations and represents the present value of profits from all future rotations.

LEV1 was chosen in this study instead of the classical land expectation value (LEV) proposed by Faustmann (1849) because the stumpage price with all its uncertainty, stand volume, regeneration cost, and interest rates are unlikely to repeat themselves from rotation to rotation forever. To explore the concept and compare with the results of B&M method, the cost of purchasing the American put option was not considered.

Given the relevant parameters shown in Table 1, the optimal rotation age under certainty will be 31 for loblolly pine and LEV1 of $4746/ha. An expiration age of 70 was chosen to ensure that it is long enough to capture the benefits of price fluctuations. To calculate the put option value, the average real stumpage price of southern pine in Louisiana from 1956 to 2015 of $71.69/m3 with a standard deviation of $27.85/m3 (Zhang and Chang 2018) was chosen as both the spot price at age 15 and strike price at age 70. As shown in Table 1 with a real interest rate of 4%, the option value calculated by the binominal tree method of Cox et al. (1979) method, at age 15 the 55-year American put option was valued at $26.11 per cubic meter and gradually declines to $0 per cubic meter at age 70. For any year t­1, ≤ 1 ≤ , the reservation price RP(t­1) can be calculated as follows:

RP(t­1) = + (2)

In equation (2) the option value per hectare equals . This amount is then divided by the stand volume at age t1 to determine the price premium per cubic meter at age t1. Add this premium to the strike price P1(T1) to arrive at the reservation price. The spot price at age t1 must exceed the reservation price to be in the money. For example, given the settings above, the reservation price at age 15 would be

RP(15)=71.69+ (26.11\*487.56)/30.52 = $488.80/m3.

This reservation price at age 15 was frighteningly high because the option value of $26.11/m3 must be multiplied by 15.97 (487.56 m3 at age 70 divided by 30.52 m3 at age 15). If the spot price at age 15 is higher than this reservation price, literally with no upside potential and 100% downside potential, an immediate timber harvest would be justified. As the stand age increases, the difference between Q(t­1) and Q(70) narrows. Over time, both the option value and the reservation price decrease. By age 70, with an option value of 0, the reservation price is the same as the target strike price of $71.69/m3. But the American put option guarantees that forestland owners and managers can sell the timber volume of the 70-year-old stand at the strike price, which the B&M method lacks. Compared with the reservation prices obtained by the B&M method, Figure 1 shows that the reservation prices obtained by the 55-year put option approach are higher everywhere than those of B&M. The gap between the two is at its widest initially, then narrows down over time until they converge at age 70.

To examine the impact of the reservation prices of these two methods on the optimal rotation age and LEV1, 50,000 simulations each were carried out. Shown in Table 2 are the average rotation age and LEV1 of the two methods. The 55-year American put option method resulted in an average optimal rotation age of 46 years and an average LEV1 of$6503/ha. The former is 50% longer than the 31 years under certainty and the latter is 37 % higher than the LEV1 of $4746/ha under certainty. When compared with the results of the B&M method, however, the former was 10.7 years longer than the 35.45 years of the B&M method and the latter is $617/ha less than the $7120/ha of the B&M method. Visual inspection of the reservation price curves of Figure 1 indicates that the reservation price curve of the 55-year American put option is tilted backward towards the end of 70 years. Not surprisingly, it results in a much longer average rotation age.

*Rolling put options*

Given that the full-length put option of 55 years resulted in very high reservation prices during its earlier years and yielded long average rotation age, rolling put of shorter durations are explored to see if the land expectation value can be enhanced while the rotation age can be reduced. Specifically, 10, 13, 15, 17, 19, 20, 21, 23, 25 and 40-year rolling put options were explored. For a rolling 10-year put, at age 15, the put option will expire at age 25, to be followed by a put option at age 16 to age 26 and so on until age 60. From age 61 until age 70, it will conclude the last 10 years with put options of 9, 8, 7, 6, 5, 4, 3, 2, 1 and 0 year. The reservation prices obtained with the rolling 10, and 20-year rolling put options are shown in Table 1 and Figure 1. The shorter rolling put options both lowered and flattened the reservation prices, which, in turn, affect the average rotation age and land expectation value.

Table 2 presents the average LEV1 and rotation age plus their standard deviations from 50,000 simulations for rolling put option of various lengths. Among them, the 15-year put rolling option resulted in the highest average LEV1 of $7094/ha with an average rotation age of 35.7 years. These results are, essentially, the same as the average LEV1 of $7120/ha and rotation age of 35.5 years obtained with the B&M method.

Interestingly, the 10-year rolling American put option results in a very flat reservation price curve. The average rotation age shortened to a surprising 29.9 years and an average LEV1 of $6846/ha. The former was even shorter than the rotation age of 31 years while the latter was 44% larger than the LEV1 of $4746/ha under certainty. Furthermore, the LEV1 with the 10-year rolling put option was only 3.5% less than that of the 15-year rolling put option. As shown in Figure 2, the land expectation value peaked with the 15-rolling put option. Longer rolling put options afterwards resulted in steadily declining land expectation value, with that of the 40-year rolling put option only about $6 higher than that of the 55-year option.

*The meaning of the rolling put option*

When a rolling put option of shorter duration is deployed, only a portion of the stand volume was covered by the put option when compared with the case of the 55-year long put option. For example, at age 15 the rolling 15-year put option will cover 178 cubic meters of the stand volume at age 30. Compared with the 487.56 cubic meters stand volume of age 70 covered by the 55-year put option, this translated into a partial coverage of 36.51%. Similarly, the rolling put option at age 20 will result in a partial coverage of 46.97%. As shown in Table 3, as stand age increases, the coverage ratio gradually increases to 100% at age 55 and thereafter. Also presented in Table 3 is the coverage ratio of the 10-year and 20-year rolling put options. These coverage ratios lead to two observations. First, rolling put options result in partial coverage which increases with stand age, eventually reaching 100%. Second, the shorter the duration of the rolling put option, the lower the coverage ratio. By choosing a shorter rolling put option, the forestland owners are expressing their level of uncertainty tolerance – the shorter the put option, the more uncertainty tolerant. Borrowing from Arrow’s (1971) terminology on risk bearing, the shorter the rolling put options, the more risk seeking.

*Discussion*

Regardless of the duration of the put options, when compared with the outcome under certainty assumption, outsourcing stumpage price uncertainty with an American put option increased the timberland value expressed as LEV1. Moreover, Figure 2 shows that the duration of the rolling put option and the average LEV1 exhibits a curvilinear relation. Since a full length 55-year put option means the forest landowner is extremely uncertainty (risk) averse, wanting to cover all downside stumpage price uncertainty. A put option with a shorter duration, on the other hand, might provide a choice for people who are more tolerant of uncertainty. In other words, the length of put option reflects forestland owner’s level of uncertainty tolerance. The curvilinear relation between the land expectation value and the length of the rolling put option indicates that a specific level of uncertainty tolerance exists to maximize LEV1,reaffirming the finding of Zhang and Chang (2018).

Even though the 15-year rolling put option produced the best financial outcome, note that between a rolling option duration of 13 and 20, their land expectation values were within $115.15/ha or 1.6% of that of the 15 years, suggesting that the length of the rolling put option makes a relatively small difference within the neighborhood of the optimal rolling put option length. Further, the surprisingly small difference in LEV1 between a rolling 10-year and 15-year put options of only $248/ha ($7094 - $6846) or 3.5% suggests that for the current example being very aggressive in terms of uncertainty tolerance carries a relatively small price. On the other hand, forestland owners may want to shun away from choosing longer rolling puts, which results in significantly lower land expectation values. Surely, additional research is required to confirm if these initial conclusions remain valid for stumpage prices with wider standard deviation or different species. Moreover, as shown in Table 4, for the 15-year rolling put option, .7% of the time timber harvests occurred before age 20; 0.78% of the time they occurred between 60 and 65 years, and .33% of the time they occurred after age 65 for a total 1.81% of the time. Thus, the financial impact of selecting the starting and ending ages of the management window is of high practical importance. Further, field forestry operations often involve annual income from hunting lease, carbon sequestration, and other payment for ecosystem services as well as annual management expenses. How such inclusions would affect the option value and consequently reservation price, harvest age and the financial outcome of management decisions awaits careful exploration.

Lastly, how the concept of applying the American put option should be implemented awaits careful exploration. Intuitively, the American put option acts like a stumpage price insurance guarding against the downside uncertainty of a specific strike price. As such, the insurance industry may find it worthwhile to offer long put options on stumpage prices to timberland owners. Given most options are short living, lasting for weeks or months, such long put options would test the limits of put options as currently understood and open the field for further theoretical investigations.

*Conclusions*

This article demonstrates that the concept of American put option can be applied effectively to outsource stumpage price uncertainty and enhance the land expectation value. The reservation prices derived with the American put options for various stand ages establish guidance for forestland owners to determine whether the spot market price is high enough to justify an immediate timber harvest. By following this approach, the forestland owners could realize higher harvest revenue and higher land expectation values to the tune of 40%, similar to the 42 and 50% increases in land expectation values for high and low sites of loblolly pine respectively of B&M. Whereas this efficacy in enhancing the land expectation value is beyond disputes, the actual figures of the land expectation value reported here are of lesser importance because they depend critically on the interest rate and other input factors used in the calculation.

With the American put option to determine the option value and reservation price, we are ushering in the era of active forest management which allows timberland owners to actively manage stumpage price uncertainty. They can explore different combinations of strike price, interest rate, the length of the management window, and the length of the rolling put options for the corresponding reservation prices, resulting in specific land expectation value and rotation length reflecting their level of uncertainty tolerance.

Once forestland owners begin actively selecting those relevant parameters, they become price setters rather than price takers. That would bring about a sea change in the stumpage market. As Gong and Löfgren (2007) have shown, its implications on timber supply and social welfare could be far reaching and need to be fully explored.

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Table 1: Reservation prices determined by the Brazee and Mendelsohn method as well as the American put option of various lengths.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | P(t)= | $71.69/m3 | Std. Dev.= | $27.85/m3 | C= | 592.8/ha |  |
|  | r= | 0.04 | LEV2= | $4,940/ha |  |  |  |
|  | Stand Volume | LEV1 | B & M | American Put 55-year | | American Put Rolling 10-year | American Put Rolling 20-year |
| Age t | Q(t)\* | Under Certainty | Reserv. Price | Option value | Reserv. Price | Reserv. Price | Reserv. Price |
| Years | m3/ha | $/ha | $/m3 | $/m3 | $/m3 | $/m3 | $/m3 |
| 15 | 30.52 | 3319 | 298.46 | 61.62 | 488.75 | 160.72 | 256.11 |
| 20 | 73.71 | 4001 | 165.79 | 61.55 | 244.20 | 124.14 | 163.94 |
| 25 | 125.10 | 4524 | 127.57 | 61.29 | 172.90 | 111.45 | 134.64 |
| 30 | 178.00 | 4739 | 112.78 | 60.78 | 142.23 | 105.44 | 121.45 |
| 31 | 188.42 | 4746 | 110.97 | 60.64 | 138.17 | 104.62 | 119.69 |
| 32 | 198.74 | 4742 | 109.38 | 60.48 | 134.56 | 103.89 | 118.11 |
| 33 | 208.95 | 4728 | 107.99 | 60.32 | 131.33 | 103.22 | 116.70 |
| 34 | 219.04 | 4705 | 106.76 | 60.13 | 128.41 | 102.62 | 115.43 |
| 35 | 228.99 | 4674 | 105.67 | 59.94 | 125.77 | 102.08 | 114.28 |
| 40 | 276.61 | 4408 | 101.65 | 59.86 | 116.39 | 99.99 | 109.89 |
| 45 | 320.40 | 4021 | 99.12 | 59.37 | 109.97 | 98.59 | 106.98 |
| 50 | 360.37 | 3572 | 97.36 | 58.01 | 104.94 | 97.60 | 104.94 |
| 55 | 396.75 | 3106 | 95.89 | 55.35 | 100.51 | 96.87 | 100.51 |
| 60 | 429.86 | 2651 | 94.02 | 51.26 | 96.33 | 96.33 | 96.33 |
| 65 | 460.02 | 2224 | 89.21 | 42.36 | 90.72 | 90.72 | 90.72 |
| 70 | 487.56 | 1833 | 71.69 | 0 | 71.69 | 71.69 | 71.69 |

\*Q(t) =5.83\*exp(12.09-52.9/t) from Brazee and Mendelsohn (1988)

Table 2: Average land expectation value in $/ha and rotation age in years based on rolling puts of different lengths

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Method | LEV1 | | Rotation age | |
|  | Average | Standard deviation | Average | Standard deviation |
|  |  |  |  |  |
| B&M | 7120 | 902 | 35.45 | 7.2 |
|  |  |  |  |  |
| 10-year rolling put | 6846 | 768 | 29.86 | 7.54 |
| 13-year rolling put | 7063 | 854 | 33.54 | 8.30 |
| 15-year rolling put | 7094 | 970 | 35.69 | 8.72 |
| 17-year rolling put | 7073 | 1092 | 37.61 | 9.03 |
| 19-year rolling put | 7018 | 1202 | 39.27 | 9.25 |
| 20-year rolling put | 6984 | 757 | 40.09 | 9.29 |
| 21-year rolling put | 6958 | 1289 | 40.71 | 9.34 |
| 23-year rolling put | 6868 | 1368 | 42.07 | 9.38 |
| 25-year rolling put | 6809 | 1414 | 42.98 | 9.30 |
| 40-year rolling put | 6518 | 1491 | 46.05 | 8.32 |
| 55-year put | 6503 | 1483 | 46.15 | 8.28 |

Table 3. Partial coverage ratios at various ages of three rolling put options

|  |  |  |  |
| --- | --- | --- | --- |
| stand age | 10-year rolling put | 15-year rolling put | 20-year rolling put |
| 15 | 0.2566 | 0.3651 | 0.4697 |
| 20 | 0.3651 | 0.4697 | 0.5673 |
| 25 | 0.4697 | 0.5673 | 0.6572 |
| 30 | 0.5673 | 0.6572 | 0.7391 |
| 35 | 0.6572 | 0.7391 | 0.8137 |
| 40 | 0.7391 | 0.8137 | 0.8817 |
| 45 | 0.8137 | 0.8817 | 0.9435 |
| 50 | 0.8817 | 0.9435 | 1.0000 |
| 55 | 0.9435 | 1.0000 | 1.0000 |
| 60 | 1.0000 | 1.0000 | 1.0000 |
| 65 | 1.0000 | 1.0000 | 1.0000 |
| 70 | 1.0000 | 1.0000 | 1.0000 |

Table 4. Stand harvest age for the 15-year rolling put

|  |  |  |
| --- | --- | --- |
| Stand Harvest Age | Frequency | |
|  |  |  |
| 20 | 349 | 0.70% |
| 25 | 4653 | 9.31% |
| 30 | 10483 | 20.97% |
| 35 | 11833 | 23.67% |
| 40 | 9830 | 19.66% |
| 45 | 6218 | 12.44% |
| 50 | 3446 | 6.89% |
| 55 | 1755 | 3.51% |
| 60 | 875 | 1.75% |
| 65 | 391 | 0.78% |
| 70 | 167 | 0.33% |

Figure 1. Reservation prices under the Brazee and Mendelsohn method, 55-year American put and rolling 10-year and 20-year American put.

Figure 2. The relationship between LEV1 and the length of the rolling put options. The first number in the box is the length of the put option and the second number the land expectation value.